

## Experimental and Theoretical Investigation of Performance of a Solar Chimney Model Part II: Model Development and Validation

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**Abstract:** A mathematical model based on the momentum, continuity and energy balance equations was developed to simulate the behavior of the air flow inside the solar chimney system. The model can estimate the power output and performance of solar chimney systems. The developed mathematical model is validated by the experimental data that were collected from small pilot solar chimney; (experiment was presented in part I). Good agreement was obtained between the experimental results and that from the mathematical model. The model can be used to analyze the solar chimney systems and to determine the effect of geometrical parameters such as chimney height and collector diameter on the power output and the efficiency of the system.

### دراسة نظرية وعملية لأداء نموذج المدخنة شمسية، الجزء الثاني: تطوير النموذج والتحقق

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**الملخص:** هذه الورقة تعرض نموذجاً رياضياً تم تطويره بناء على قوانين حفظ الكتلة والطاقة والزخم الحركي وذلك لدراسة سلوك انسياب الهواء خلال منظومة المدخنة الشمسية مع استخدام هذا النموذج الرياضي لتقييم أداء المنظومة وتقدير القدرة الناتجة من منظومة المدخنة الشمسية. تمت مقارنة نتائج النموذج الرياضي بالنتائج التجريبية الواردة في الجزء الأول من هذه الورقة والمتحصل عليها من المدخنة الشمسية التجريبية التي صممت وبنيت في كلية الهندسة في مدينة صبراتة - ليبيا. ومن خلال المقارنة بين نتائج النموذج الرياضي المطور وتلك المتحصل عليها من المدخنة الشمسية التجريبية تبين ان هناك توافقاً جيداً بينهما. وهذا النموذج الرياضي الذي تم تطويره في هذه الورقة يمكن استخدامه في تحليل نظم المدخنة الشمسية وتحديد أثر المعايير الهندسية مثل قطر المجمع الشمسي وارتفاع المدخنة الشمسية في إنتاج الطاقة وكفاءة النظام.

**Keywords:** Solar energy, Solar chimney; Buoyancy effect; Draft tower; Renewable energy.

## 1. INTRODUCTION

Solar chimney power plant (SCPP) is a novel technology for electricity production from solar energy. The SCPP consists of a greenhouse roof collector and updraft chimney that is located at the center of the greenhouse roof collector. The greenhouse roof collector is usually made of plastic sheet or glass plate which traps solar radiation and elevates the air temperature. The chimney is used to direct and vent the hot air through the wind turbine. The wind turbine is used to convert the air kinetic energy into mechanical work. No full scale of solar chimney power plant (SCPP) has been operated to date; however many projects have been investigated in different parts of the world. The first outstanding solar chimney project was the prototype erection in 1982 in Manzanares, 150 km south of Madrid, Spain. The chimney height was 195 m and its diameter 10 m. The collector area was 46,000 m<sup>2</sup> [1]. Regardless of its dimensions, this prototype was considered as a small-scale experimental model. As this model was not intended for power generation, the peak power output was about 50 kW. The first known attempt for solving Navier-Stokes and energy equations for the natural laminar convection in steady state, predicting its thermo-hydrodynamic behavior using CFD (Computational Fluid Dynamics) for convective flow in a SCPP was conducted by Bernardes, et al [2]; the approach employed “Finite Volumes Method” in generalized coordinates allowing for a detailed visualization of the effects of geometrical and operational characteristics. A set of differential equations for SCPP were deduced and integrated by Padki and Sherif [3]. Gannon and von Backström [4] introduced a study including chimney friction, turbine and exit kinetic energy losses in the analysis. Schlaich, et al. [5] presented a simplified theory, practical experience, and economy of solar updraft towers.

The objective of this paper is to develop a model that simulates the behavior of the solar chimney system and to estimate the available power that can be converted to electrical power by the system.

## 2. MATHEMATICAL MODEL

In the solar chimney the heat energy is provided by the solar collector, and flow rate is generated by buoyancy force and the chimney’s stack effect. According to this principle, however, the driving force provided by the chimney could overcome the pressure drop in the system and ensure the functionality of the collector. In addition, the suction of hot air flowing through the chimney could be used in power generation. The basic operation principles behind the solar chimney are summarized as follows:

1. Heat is received by the solar collector due to solar irradiation from the sun.
2. Air is heated in the solar collector mostly by the heat convection from the absorber.
3. Hot air then moves into the chimney due to the buoyancy and stack effect induced in the system. The three components of the solar chimney system collector, chimney and turbine are simulated and the mathematical model is presented below.

### 2.1 Assumptions

To simplify the analysis of the solar chimney system, the following assumptions were made.

1. One-dimensional heat transfer through the system layers in the (z-direction).
2. There is no heat transfer in the direction of the flow, the energy is transferred in the flow direction by mass transfer.
3. The heat losses from the collector edges are negligible.
4. Properties of the absorber and insulation are independent of temperature variation.
5. The sky can be considered as a black body for long-wave length radiation at an equivalent sky temperature.
6. Dust and dirt on the collector are negligible.
7. Axis-symmetric flow of the air inside the

collector

8. The main direction of flow in the collector is the direction, no flow in z and  $\theta$  directions.
9. Constant height of collector cover, i.e. radial inward flow is considered between two parallel plates.
10. The heat losses through the walls of the chimney are neglected; and the flowing humid air is considered as a mixture of two ideal gases.
11. The upwind flow is due to the buoyancy force only.
12. The flow in the collector is considered as a laminar flow.
13. Convection heat transfer in the collector is considered as the free convection mode.

Under the above assumptions the continuity equation for each segment of the collector becomes:

$$(\rho u A)_{r+dr} = (\rho u A)_r, \quad \dot{m}_{coll.} = \dot{m}_{ch.}$$

$$\dot{m}_{chem.} = \rho_{chim.} V_{chim.} A_{chim.}$$

Where:  $\dot{m}_{coll.}$ ,  $\dot{m}_{chim.}$ ,  $\rho_{chim.}$ ,  $V_{chim.}$  and  $A_{chim.}$

are the air mass flow rate through the collector, the air mass flow rate through the chimney, density of air through the chimney, velocity of air through the chimney and cross-sectional area of the chimney.

## 2.2 Solar Collector Analysis

The solar collector provides the main natural source of heat energy in the solar system. As the name suggests, the solar collector collects heat energy from solar irradiation via the absorber layer and transfers it to the working fluid (air). The main components of the basic solar collector are: transparent cover and the absorber layer. The heat losses through the solar collector are due to conduction, convection, and radiation. Conduction losses occur through the layer underneath of the absorber (bottom loss) and through the edges of the collector. Heat loss through the cover (top loss) is due to convection and radiation to the ambient. The solar

chimney collector is considered as a control volume containing a number of radial sections as indicated in Figures 1 and 2. Each section consists of four nodes at different media (cover, air gap, absorber, ground or insulator underneath of the absorber). The nodes are distributed in a perpendicular plane to the air flow direction (see Figure 2).

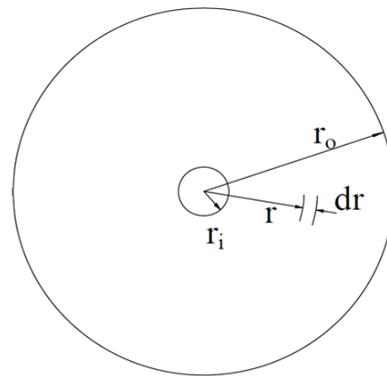


Figure (1). Top view of the solar collector

The useful energy  $Q_u$  which may be collected by the collector of area  $A_{coll}$  is equal to the difference between the absorbed solar radiation and the thermal energy loss. Therefore,

$$Q = A_{coll} [S_1 - U_t(T_c - T_{amb}) - U_b(T_{ab} - T_{soil})] \dots\dots\dots(1)$$

Where,  $U_t$  is the top loss coefficient ( $W/m^2 K$ ),  $U_b$  is the back loss coefficient ( $W/m^2 K$ ),  $T_c$  is the cover temperature,  $T_{amb}$  is the outside air temperature,  $T_{ab}$  is the absorber temperature and  $T_{soil}$  is the soil temperature under the absorber. The thermal efficiency  $\eta_{cool}$ , which is defined as the ratio of the net useful energy collected by collector,  $Q_u$  to the total incident solar radiation and expressed by the Hottel-Whillier-Bliss Equation [6] as.

$$\eta_{coll} = \frac{Q_u}{Q_n}, \quad Q_n = I \times A_{coll} \dots\dots\dots(2)$$

Where,  $I$  is the intensity of solar radiation ( $W/m^2$ ),  $A_{coll}$  is the collector surface area ( $m^2$ ) and  $Q_{in}$  is the amount of solar radiation received by the collector at its cover.

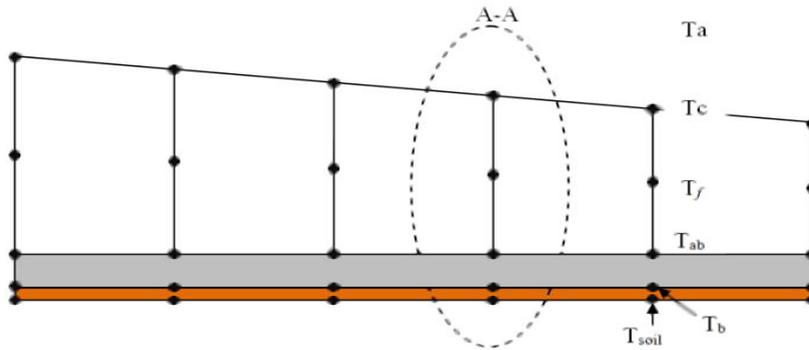


Figure (2). Nodes at different media of the solar collector

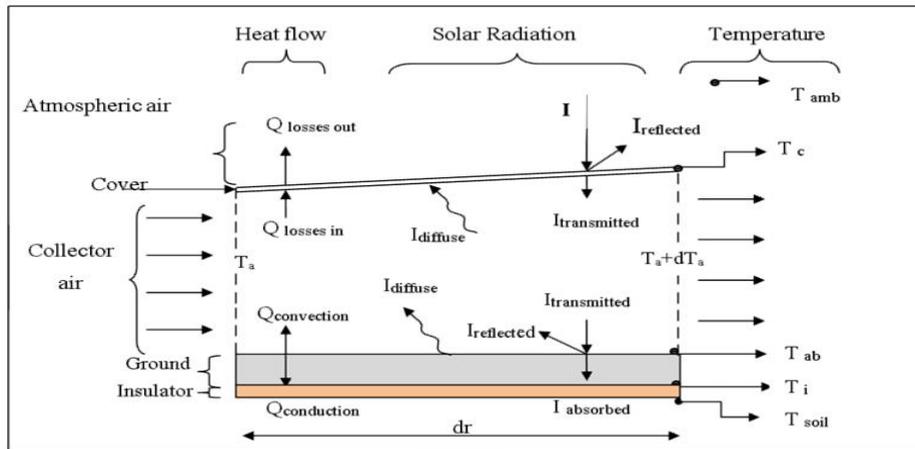


Figure (3). A section from solar collector showing temperatures and heat transfers through control volume

As shown in Figure 3, a part of solar radiation  $I$ , is reflected back to the sky, another part is absorbed by the cover and the rest is transmitted through the cover to the absorber as a short wave radiation. The percentage of the solar rays transmitted through the transparent cover of the collector is called the transmissivity of the cover ( $\tau_c$ ), and the percentage being absorbed by the cover is the absorptivity of the cover ( $\alpha_c$ ). Thus the intensity of solar radiation absorbed by the absorber layer can be expressed as:

$$S_i = I \times (\tau_c) \times \alpha_{ab} \dots\dots\dots (3)$$

Where;  $\alpha_{ab}$  is the absorptivity of the absorber.

When the absorber collector absorbs thermal

radiation, its temperature becomes higher than that of the surrounding. This leads to transferring an amount of heat to the air inside the collector by convection and another amount of heat to the cover by radiation, while a part of heat is lost by conduction to the ground underneath the absorber. The rest of energy is stored inside the absorber. The rate of heat loss  $Q_{out}$ , depends on the collector overall heat transfer coefficients ( $U_t$ ,  $U_b$ ) and the temperature differences.

$$Q_{out} = \{U_t (T_c - T_a) + U_b (T_{ab} - T_{soil})\} \times A_{coll} \dots\dots\dots (4)$$

Where  $U_t$  is the top overall heat losses coefficient ( $W/m^2 K$ ) and  $U_b$  is the back overall heat losses coefficient ( $W/m^2 K$ ).

**(a) The cover**

The small thickness of the cover makes it reasonable to consider a uniform temperature and negligible heat storage within the cover. The energy balance at the cover control volume of thickness ( $\delta c$ ) and area of ( $2\pi r dr$ ) can be presented as:

$$[h_w(T_{amb} - T_c) + h_{c-a}(T_a - T_c) + h_{r_{ab-c}}(T_{ab} - T_c) + h_{r_{c-s}}(T_s - T_c) + S_2] \dots \dots \dots (5)$$

Where,  $S_2 = I \alpha_c$  and  $h_w$  may be evaluated by using McAdams correlation for the convection coefficient of the wind effect [6] as,

$$h_w = 5.7 + 3.8 V_w$$

Where,  $h_w$  is the convection heat transfer coefficient between cover and ambient ( $W/m^2 K$ ),  $V_w$  is the wind velocity,  $h_{c-a}$  is the convection heat transfer coefficient between cover and air in the collector ( $W/m^2 K$ ),  $h_{r_{ab-c}}$  is radiation heat transfer coefficient between absorber and cover ( $W/m^2 K$ ),  $h_{r_{c-s}}$  is the radiation heat transfer coefficient between cover and sky. Equation (5) can be written as,

$$T_c = \frac{(h_w T_{amb} + h_{c-a} T_a + h_{r_{ab-c}} T_{ab} + h_{r_{c-s}} T_s + I \alpha_c)}{(h_w + h_{c-a} + h_{r_{ab-c}} + h_{r_{c-s}})} \dots (6)$$

$$[h_{c-a}(T_c - T_a) + h_{ab-a}(T_{ab} - T_a)](2\pi r dr) = dQ_u \dots \dots (7)$$

The mean temperature at each section is evaluated as,

$$T_a = \frac{T_{a_{out}} + T_{a_{in}}}{2} \dots \dots \dots (8)$$

Where,  $T_{a_{in}}$  and  $T_{a_{out}}$  are the inlet and outlet air temperatures at the collector sections respectively. The useful rate of heat transferred to the moving air stream can be written in terms of the mean fluid and inlet temperature as,

$$dQ_u = 2 \dot{m} c_p (T_a - T_{a_{in}}) \dots \dots \dots (9)$$

Then Equation (7) can be written as;

$$T_a = \frac{(h_{c-a} T_c + h_{ab-a} T_{ab} + \frac{\dot{m} C_p}{\pi r dr} T_{a_{in}})}{(h_{c-a} + h_{ab-a} + \frac{\dot{m} C_p}{\pi r dr})} \dots \dots (10)$$

**(b) The air between the cover and the absorber**

Consider a control volume on the air gap inside the solar collector. The rate of heat balance of the air flows through the control volume can be written as,

**(c) The absorber**

When the absorber temperature is higher than that of the surrounding, some amount of the heat transfers to the air inside the collector by convection and some heat is transferred by radiation to the cover of the collector, and another heat segment is lost by conduction to the ground underneath the absorber. The rest of energy is stored inside the absorber. The energy balance at differential control volume of the absorber of thickness ( $\delta_{ab}$ ) and area of ( $2\pi r dr$ ) can be presented as:

$$h_{ab-a}(T_a - T_{ab}^t) + h_{r_{c-ab}}(T_c - T_{ab}^t) + U_{Gr-ab}(T_{soil} - T_{ab}^t) + S_2 = \delta_{ab} \rho_{ab} c (T_{ab}^{t+\Delta t} - T_{ab}^t) \dots \dots \dots (11)$$

Then,

$$T_{ab}^{t+\Delta t} = T_{ab}^t \left[ 1 + \frac{(h_{ab} + h_{r_{ab-c}} + U_{Gr-ab})}{\delta_{ab} \rho_{ab} c} \right] + \left[ \frac{(h_{ab-c} T_a + h_{r_{ab-c}} T_c + U_{Gr-ab} T_{soil} + S_2)}{\delta_{ab} \rho_{ab} c} \right] \dots \dots \dots (12)$$

where:  $S_2 = I \tau_c \alpha_{ab}$ ,  $U_{Gr-ab} = \frac{k_i k_{soil}}{\delta_i k_{soil} + \delta_{soil} k_i}$

Where  $U_{Gr-ab}$  is over all heat transfer coefficient under the absorber ( $W/m^2 .K$ ),  $T_{ab}$  is the absorber temperature (K),  $T_{soil}$  is the soil temperature under

insulation layer at  $\forall$  (K),  $\delta_{ab}$  is the absorber thickness (m),  $\delta_i$  is the insulator thickness (m),  $\delta_{soil}$  is the soil thickness (m),  $k_{soil}$  is thermal conductivity of the soil (W/m K),  $k_{ab}$  is thermal conductivity of the absorber (W/m K),  $k_i$  is thermal conductivity of the insulator (W/m K) and the superscript (t) is the time at which

the absorber temperature is considered.

Estimating the convection heat transfer coefficients is a very complex process. Therefore simplified correlations were used for this purpose. Table 1 shows a sample of these correlations [7, 8].

Free convection $Nu_m = 0.54 Ra^{1/4}$  $Nu_m = 0.14 Ra^{1/3}$	$10^4 \leq Ra < 10^7$ , upper or lower heated  horizontal surface [8]  $10^7 \leq Ra \leq 10^{11}$ , upper or lower Heated  horizontal surface, [8]	
State	$T_c < T_a$	$T_c > T_a$
Heat convection coefficient between cover and air in the collector for laminar flow [7].	$1.32 \left( \frac{T_a - T_c}{dr^*} \right)^{1/4}$	$0.59 \left( \frac{T_c - T_a}{dr^{*2}} \right)^{1/4}$
State	$T_{ab} > T_a$	$T_{ab} < T_a$
Heat convection coefficient of absorber and air in the collector for laminar flow [7].	$1.32 \left( \frac{T_{ab} - T_a}{dr^*} \right)^{1/4}$	$0.59 \left( \frac{T_a - T_{ab}}{dr^{*2}} \right)^{1/4}$

Table (1) Correlations for estimating convection heat transfer coefficients [7, 8]

\*  $dr$  is the collector section length in the flow direction (m).

To determine the radiation heat transfer coefficients, the conventional relation is used as:

$$h_{r_{ab-c}} = \frac{\sigma(T_{ab}^2 + T_c^2)(T_{ab} + T_c)}{1/\epsilon_{ab} + 1/\epsilon_c - 1} \dots\dots (13)$$

The radiation heat transfer coefficient between the cover and atmosphere (sky) is given as,

$$h_{r_{c-s}} = \frac{\sigma \epsilon_c (T_c^2 + T_s^2)(T_c + T_s)(T_c - T_s)}{(T_c + T_{amb})} \dots\dots (14)$$

The sky temperature is given by Swinbank in [6] as;

$$T_s = 0.0552 * T_{amb}^{1.5}$$

Where  $T_{amb}$  is the ambient temperature (K).

#### d. Chimney analysis

The chimney converts the heat produced by the collector into kinetic energy and potential energy. Thus, air density difference which is caused by temperature rising in the collector works as a driving force. The pressure difference  $\Delta P_{tot}$  produced between chimney base and the ambient may be calculated as,

$$\Delta P_{tot} = \int_0^{H_{chim}} (\rho_{amp} - \rho_{chim}) g \dots\dots\dots (15)$$

$$\Delta P_{tot} = (\rho_{chim} - \rho_{amp}) g H_{chim} \dots\dots\dots (16)$$

also  $\rho_{chim} = \frac{P_{chim}}{RT_{chim}}$ ,  $\rho_{amp} = \frac{P_{amp}}{RT_{amp}}$  and  $P_{chim} \cong P_{amp}$

Then,

$$\Delta p_{tot} = \rho_{chim} * g * H_{chim} * \frac{\Delta T}{T_{amb}} \dots\dots\dots (17)$$

Where,  $H_{chim}$  is the chimney height (m),  $\rho_{amp}$  is the air density at ambient temperature ( $kg/m^3$ ) and  $\rho_{chim}$  is the air density through the chimney ( $kg/m^3$ ). The total pressure difference causes the flow of air through the chimney.  $\Delta P_{tot}$  is equal to the sum of the static  $\Delta P_s$ , dynamic  $\Delta P_d$  pressure differences and the pressure drop due to the friction.

$$\Delta p_{tot} = \Delta p_s + \Delta p_d + \Delta p_{friction} \dots\dots\dots (18)$$

$$\Delta p_d = \frac{1}{2} \rho_{chim} V^2_{chim}$$

Where,  $\Delta P_s$  is the static pressure difference drops at the turbine, when the system is without a turbine, then,  $\Delta P_s \cong 0$

Now introducing the pressure drop through the chimney due to friction loss  $\Delta P_{friction}$ , which is direct proportion to the kinetic energy per unit volume and chimney height  $H_{chim}$  and inverse proportion to the hydraulic diameter of the chimney  $D_{chim}$ , then;

$$\Delta p_{friction} = \frac{1}{2} f \frac{H_{chim}}{D_{chim}} \rho_{chim} V^2_{chim} \dots\dots\dots (19)$$

Where  $f$  is the dimensionless ‘‘Darcy friction factor’’.

From Equation (19), the pressure drop due to the friction losses is proportional to the kinetic energy per unit volume by the dimensionless term (  $f \frac{H_{chim}}{D_{chim}}$  ) which can be denoted by  $\xi$ . Then

Equation (19) can be written as:

$$\Delta p_{friction} = \xi \frac{1}{2} \rho_{chim} V^2_{chim} \dots\dots\dots (20)$$

$$\Delta p_{tot} = \frac{1}{2} \rho_{chim} V^2_{chim} + \xi \frac{1}{2} \rho_{chim} V^2_{chim} \dots\dots (21)$$

$$P_{tot} = \eta_{chim} * Q_u = \frac{g * H_{chim}}{c_p * T_{amb}} * \rho_{chim} * c_p * V_{chim} * \Delta T * A_{chim} \dots\dots\dots (28)$$

$$\Delta p_{tot} = \frac{1}{2} (1 + \xi) \rho_{chim} V^2_{chim} \dots\dots\dots (22)$$

Using Equation (17) in Equation (22) then,

$$V_{chim} = \sqrt{\frac{2 * g * H_{chim} * \Delta T}{(1 + \xi) T_{amb}}} \dots\dots\dots (23)$$

The power  $P_{tot}$  produced from the flow when the  $\Delta p_s = 0$  can be calculated as,

$$P_{tot} = \Delta p_{tot} * V_{chim} * A_{chim} \dots\dots\dots (24)$$

From which the efficiency of the chimney  $\eta_{chim}$

can be found as:

$$\eta_{chim} = \frac{P_{tot}}{\dot{Q}} \dots\dots\dots (25)$$

Where

$$\dot{Q} = m c_p (\Delta T)$$

Actual subdivision of the pressure difference into a static and a dynamic component depends on the energy taken up by the turbine. Without turbine, a maximum flow speed of  $V_{chim}$  is achieved and the whole pressure difference is used to accelerate the air and is thus converted to kinetic energy.

$$P_{tot} = \frac{1}{2} (1 + \xi) \dot{m} V^2_{chim} \dots\dots\dots (26)$$

By substituting Equation (17) in (24) and then in Equation (25), the chimney efficiency can be written as;

$$\eta_{chim} = \frac{g * H_{chim}}{c_p * T_{amb}} \dots\dots\dots (27)$$

This simplified representation explains one of the basic characteristics of the solar chimney, which is that the chimney efficiency is fundamentally dependent only on its height. Then the total power resulting from the air flow can be found as;

or

$$P_{tot} = \frac{g * H_{chim}}{T_{amb}} * \rho_{chim} * V_{chim} * \Delta T * A_{chim} \dots\dots\dots (29)$$

### e. Turbine Analysis

The heat flow produced by the collector is converted into kinetic energy and potential energy through the chimney. Due to the air density difference, the lighter column of air in the chimney produces pressure difference  $\Delta p_{tot}$  which is equal,

$$\Delta p_{tot} = \Delta p_{tur} + \frac{1}{2} \rho_{chim} V^2_{chim} + \xi \frac{1}{2} \rho_{chim} V^2_{chim} \dots\dots\dots (30)$$

If the turbine extracts a fraction ( $x_{tm}$ ) of the total driving potential  $\Delta p_{tot}$  then;

$$\Delta p_{tur} = x_{tm} * \Delta p_{tot}$$

Equation (30) can now be written as,

$$\Delta p_{tot} = x_{tm} * \Delta p_{tot} + \frac{1}{2} \rho_{chim} V^2_{chim} + \xi \frac{1}{2} \rho_{chim} V^2_{ch} \dots\dots\dots (31)$$

The air velocity through the chimney becomes:

$$V_{chim} = \sqrt{\frac{2 * \Delta p_{tot} (1 - x_{tm})}{\rho_{chim} (1 + \xi)}} \dots\dots\dots (32)$$

Using Equation (17) in Equation (35) the result is,

$$V_{chim} = \sqrt{2 * g * H_{chim} \frac{\Delta T}{T_{amb}} * \frac{(1 - x_{tm})}{(1 + \xi)}} \dots\dots\dots (33)$$

The theoretical useful power  $P_{tur}$  at the turbine becomes:

$$P_{tur} = V_{chim} * A_{chim} * \Delta p_{tur} \dots\dots\dots (34)$$

The power of the turbine takes its maximum value when:

$$\frac{dP_{tur}}{dV} = 0$$

$$\frac{dP_{tur}}{dV} = 0 = \Delta p_{tot} - (1 + \xi) \frac{3}{2} \rho_{chim} V^2_{chim} \dots\dots\dots (35)$$

$$V_{chim} = \sqrt{\frac{2}{3} * \frac{\Delta p_{tot}}{\rho_{chim} (1 + \xi)}} \dots\dots\dots (36)$$

Comparing Equation (32) with Equation (36), the maximum power can be gotten when  $x_{tm} = 2/3$

And the volumetric flow rate can be estimated as:

$$\dot{V} = V_{chim} * A_{chim} \dots\dots\dots (37)$$

Then Equation (34) can be written as:

$$P_{tur} = \dot{V} * \Delta p_{tur} \dots\dots\dots (38)$$

The maximum power is achieved when two thirds of the total pressure difference is utilized by the turbine and can be expressed as:

$$P_{tur,max} = \frac{2}{3} \eta_{coil} * \eta_{chim} * A_{coil} * I \dots\dots\dots (39)$$

$$P_{tur,max} = \frac{2}{3} * \eta_{coil} * \frac{g}{c_p * T_{amb}} * H_{chim} * A_{coil} * I \dots\dots\dots (40)$$

The maximum electrical power from the solar chimney is obtained by multiplying Equation (40) by turbine efficiency that contains blade, transmission and generator efficiency.

$$P_{electric} = P_{tur,max} * \eta_{tur} \dots\dots\dots (41)$$

### 3. NUMERICAL SOLUTION PROCEDURE

The proposed solution method was implemented by utilizing the MATLAB software. The theoretical model assumed that for a small collector, the temperatures of the “boundaries” surrounding air streams are uniform. The collector was divided into equal sections. The inlet air temperature of the first section is equal to the ambient temperature. Heat transfer and loss coefficients are evaluated according to the initially guessed values of cover, air, absorber layer, insulation layer temperatures and mass flow rate of air. An iterative process for the energy balance equations is then created until the convergence of all temperatures occurs. The inlet air temperature of the second section is set equal to the outlet temperature of the first section. The iterative procedure is repeated until the temperatures are predicted for all sections of the collector. At the end of the iteration, the program calculates the outlet temperatures of the air streams at the outlet of the last section of the collector that representing the temperature at the chimney inlet while the temperature at the chimney outlet is equal to the ambient temperature. Now the buoyancy force and the mass flow rate through the chimney can be calculated. By this repetitive and iterative process,

all required parameters including the temperatures throughout the entire collector, the mass flow rate, and the power generated by the turbine can be obtained for each time step.

The model was programmed to simulate the overall thermal system behavior and evaluate;

- 1- Temperature distribution of the collector in any layer at any section.
- 2- Air velocity distribution in the collector and air velocity through the chimney.
- 3- Mass flow rate through the chimney.
- 4- Efficiencies of the collector, the chimney and overall solar chimney system.
- 5- Total pressure drop of the system.
- 6- Electrical power output.

## 4. RESULTS AND DISCUSSION

### 4.1 Model validation

The experimental data were presented in Part-I of this paper [9] and that from reference [10] are used to validate the mathematical model. Good agreement was obtained between the experimental results and the results from the developed mathematical model. The comparisons between the results are illustrated in Figures 5 to 8. The solid lines indicate the results from the model and the symbols present the experimental results. It should be noted that the input data used in the model are the same as those in the experiment. In general, the results from the model show good agreement between the experimental results and those predicted by the model. However, some differences between the measured and the predicated results do exist due to the assumptions and idealizations of the analysis.

### 4.2 Comparison of theory with experiment

Four days with different ambient temperatures and different values of solar radiation are considered for comparison (see Figure 4).

The comparison covered air velocity through the chimney and temperatures at some specified points inside the collector and at the entrance of the chimney. The temperatures and velocities comparison is presented in Figures 5 to 8 .

Figures 5 & 6 show the comparison between

the mathematical model results and those from experiment. Figure 5 presents the temperatures of the absorber at some points in the collector, and Figure 6 presents the temperature at the chimney entrance. Any increase in the intensity of solar

radiation leads to an increase in the temperature of the absorber which in sequence leads to an increase in air temperature inside the collector. Figure 6 indicates that the temperature at chimney inlet reaches its maximum value in the afternoon.

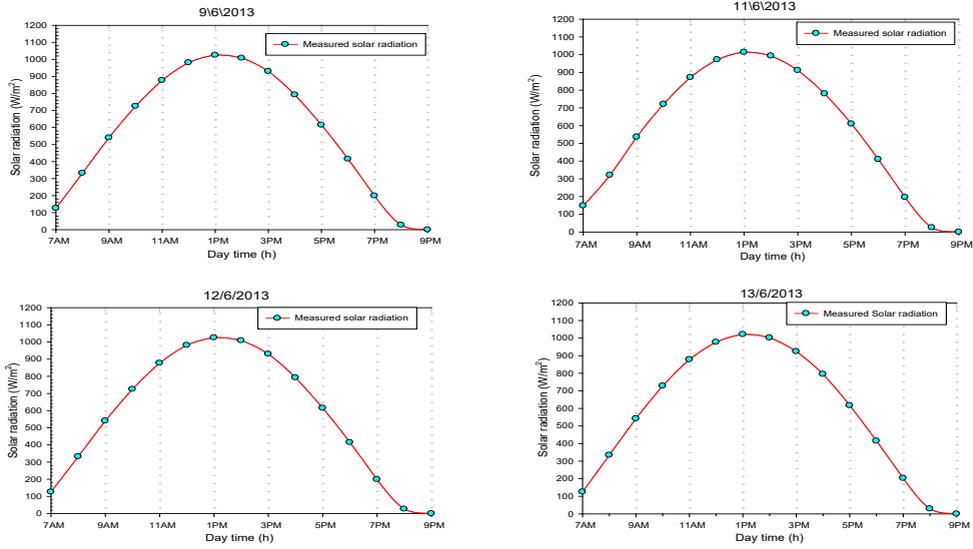


Figure (4). The Measured solar-radiation

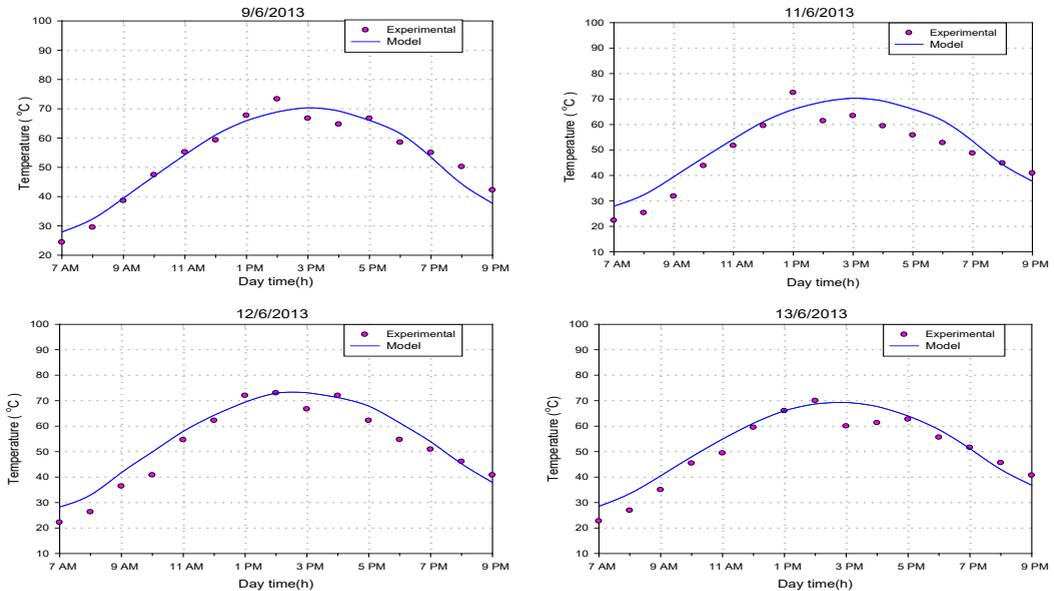


Figure (5). Comparison between the results from the model and those from experiment (Absorber temperature at collector center)

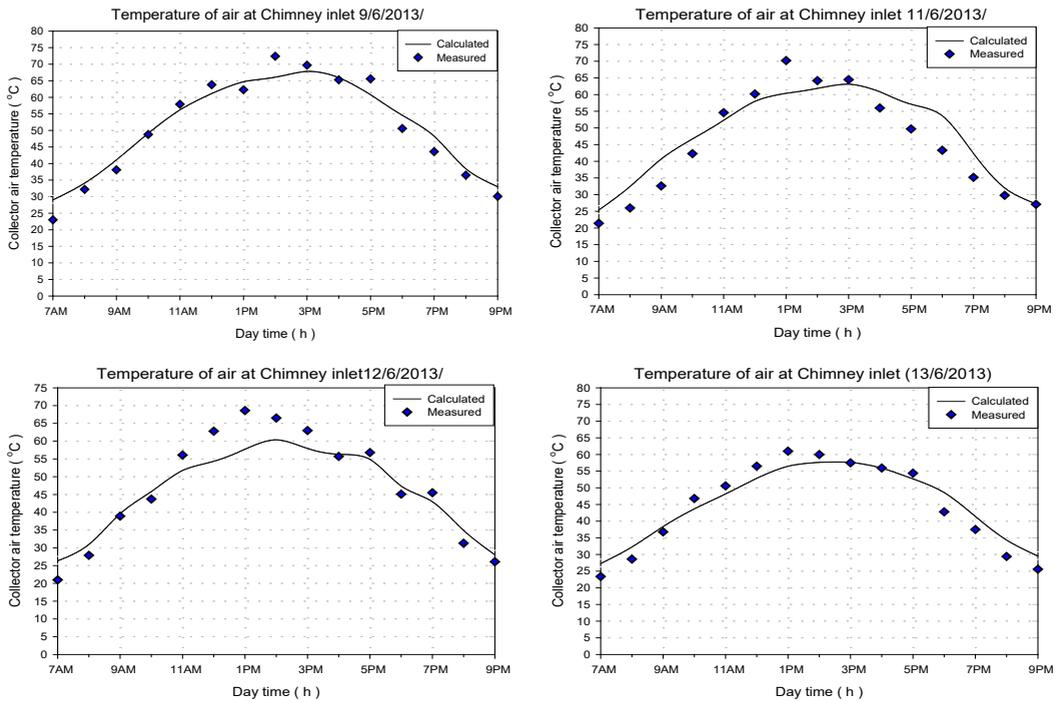


Figure (6). Comparison between the results from the model and those from experiment (Air temperatures at chimney inlet)

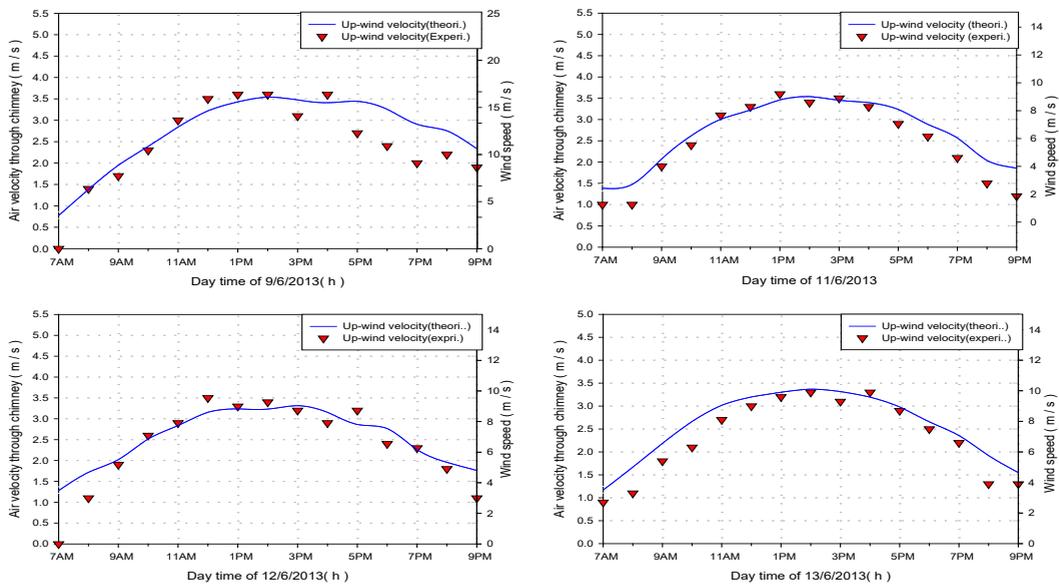


Figure (7). Comparison between the results from the model and those from experiment (Air velocity at chimney inlet)

The graphs of Figure 8 show air temperature distribution at different sections throughout the collector. The minimum values of the temperatures were recorded in the morning and reached their maximum values at noon when the solar irradiance

reaches its maximum value. The figure also indicates the maximum values of air temperatures in the middle of the collector at the entrance of the chimney.

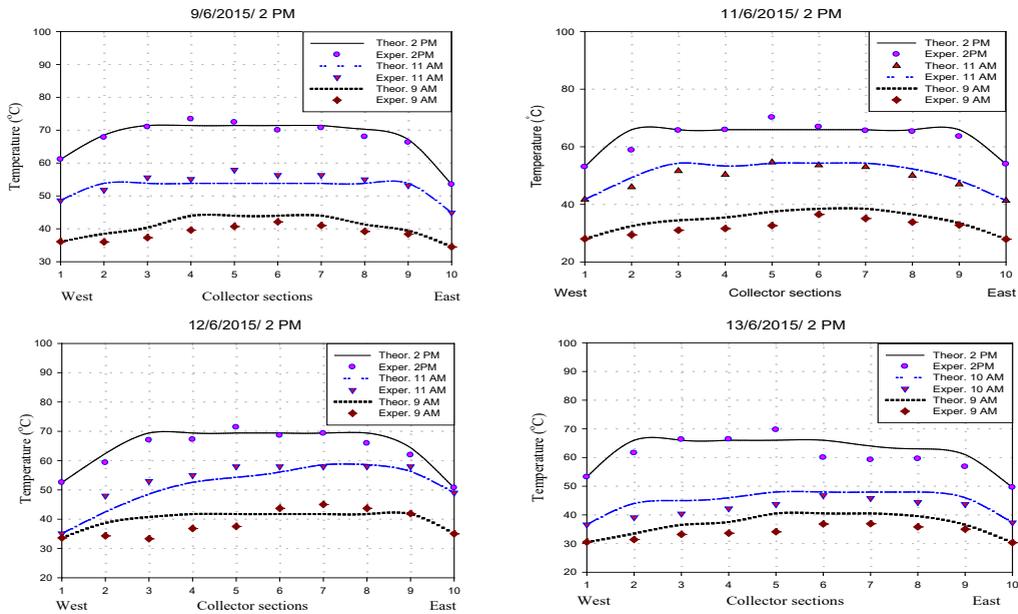


Figure (8). Comparison of air temperature distribution in the collector at different time intervals

## 5. CONCLUSIONS

This work presents an analytical model to predict the performance of solar chimney power plant. The model showed good agreement with that from the experiment. However, some differences do exist due to the assumptions and idealizations of the analysis. From the results, several conclusions may be made as follows:

1. The air speed peak is directly related to the temperature difference between collector internal temperature and the ambient temperature.
2. Temperature difference between the collector air

outlet and the ambient can reach high values during summer time in Libya, which generate an air flow that can drive a wind turbine and produce electricity through the generator. This means that solar chimney system may be operated to produce electricity in Libya.

3. The performance of solar chimney at noon hours is higher than that in the morning and evening hours.
4. The floor of solar collector stores part of thermal energy during the day and releases it during the night; therefore, using absorbers with high thermal capacity in solar chimney power plants is important to produce electricity after sunset.

## 6. NOMENCLATURE

A	Area (m <sup>2</sup> )	$S_2$	Solar radiation absorbed by absorber (W/m <sup>2</sup> )
C	Specific heat (J/kg K)	T	Temperature (K & °C)
D	Diameter (m)	U	Heat losses coefficient (W/m <sup>2</sup> K)
dr	Incremental radius (m)	V	Air velocity through chimney (m/s)
$f$	Darcy friction factor	$\dot{V}$	Volumetric flow rate (m <sup>3</sup> /s)
g	Gravitational acceleration (m/s <sup>2</sup> )	$\dot{m}$	Mass flow rate (kg/s)
H	Height (m)	$x_{tm}$	Turbine pressure extraction factor
h	Convection heat transfer coefficient (W/m <sup>2</sup> K)	<b>Greek symbols:</b>	
$h_r$	Radiation heat transfer coefficient (W/m <sup>2</sup> K)	$\alpha$	Absorptance.
I	Global horizontal irradiance (W/m <sup>2</sup> )	$\Delta$	Differential.
K	Thermal conductivity (W/mK)	$\varepsilon$	The emissivity.
Nu	Nusselt number	$\eta$	The efficiency.
Q <sub>i</sub>	Rate of energy input (W)	$\rho$	Density (kg/m <sup>3</sup> ).
Q <sub>u</sub>	Rate of useful energy output(W)	$\tau$	The transmittance of the cover.
r	Radius (m)		

## 7. REFERENCES

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